Precalculus and Trigonometry

NUMBER SENSE AND OPERATIONS INDICATORS

PCT.N.1. Define and conduct operations on complex numbers, in particular, addition, subtraction, multiplication, and division. Relate the system of complex numbers to the systems of real and rational numbers.

PCT.N.2. Plot complex numbers using both rectangular and polar coordinates systems. Represent complex numbers using polar coordinates, i.e., $a + bi = r(\cos\theta + i\sin\theta)$.

PCT.N.3. Apply DeMoivre's theorem to multiply, take roots, and raise complex numbers to a power.

Example: Use the fact that $e^{i\pi} = \cos\theta + i\sin\theta$ *to verify the really useful trigonometric identity*

$$\sin(A + B) = \sin A \cos B + \cos A \sin B$$

(See also PCT.N.1, PCT.N.2, PCT.P.2, PCT.P.9, PCT.P.10)

Example: Evaluate $e^{i\pi}$.

PATTERNS, RELATIONS, AND ALGEBRA INDICATORS

PCT.P.1. Relate the number of roots of a polynomial to its degree. Solve quadratic equations with complex coefficients, including use of completing the square.

Example: Find an equation for a polynomial with the following roots:

$+1, -1, +i, -i$.

(See also PCT.N.1)

Example: Explain why the graph of a cubic polynomial cannot cross the x-axis 4 times.

Example: Is it possible for a fourth-order polynomial to have three real solutions and one complex solution? Why or why not?

PCT.P.2. Demonstrate an understanding of the trigonometric functions (sine, cosine, tangent, cosecant, secant, and cotangent). Relate the functions to their geometric definitions.

PCT.P.3. Use matrices to solve systems of linear equations. Apply to the solution of everyday problems.

Example: Give the matrix form of the following set of equations and use Gaussian elimination to solve the system.

$2x + 4y = 10$

$3x + 2y = 7$

$$\begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 10 \\ 7 \end{bmatrix}$$

PCT.P.4. Given algebraic, numeric, and/or graphical representations, recognize functions as polynomial, rational, logarithmic, or exponential.

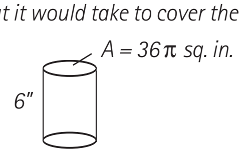
PCT.P.5. Combine functions by composition, as well as by addition, subtraction, multiplication, and division.

PCT.P.6. Identify whether a function has an inverse and when functions are inverses of each other; explain why the graph of a function and its inverse are reflections of one another over the line $y = x$.

PCT.P.7. Identify maximum and minimum values of functions. Apply to the solution of problems.

Example: A right circular cylindrical can is 6 inches high and the area of its top is 36π *square inches. What is the minimum number of square inches of construction paper that it would take to cover the lateral surface of the can?*

(See also PCT.P.7)



PATTERNS, RELATIONS, AND ALGEBRA INDICATORS (CONTINUED)

PCT.P.8. Describe the translations and scale changes of a given function f(x) resulting from substitutions for the various parameters a, b, c, and d in $y = a\ f(b\ (x + \frac{c}{b})) + d$. In particular, describe the effect of such changes on polynomial, rational, exponential, and logarithmic functions.

Example: The graph of $y = x^2$ *is a parabola with vertex at (0,0) that opens in the positive y direction. What does the graph for* $y = 3 - (x+2)^2$ *look like? Sketch the graph.*

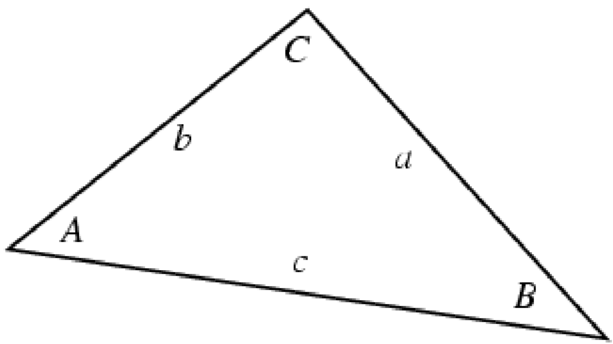
(See also PCT.P.5)

PCT.P.9. Derive and apply basic trigonometric identities i.e., $\sin^2\theta + \cos^2\theta = 1$, $\tan^2\theta + 1 = \sec^2\theta$ and the laws of sines and cosines.

Example: Use the fact that $\sin^2\theta + \cos^2\theta = 1$ *to derive the formula* $\tan^2\theta + 1 = \sec^2\theta$

(See also PCT.P.2)

Example: Use the labels on the triangle below. Find the length c *if* a =4, b = 2, *and the angle* C = 100 *degrees.*



(See also PCT.G.1)

PCT.P.10. Demonstrate an understanding of the formulas for the sine and cosine of the sum or the difference of two angles. Relate the formulas to DeMoivre's theorem and use them to prove other trigonometric identities. Apply to the solution of problems.

PCT.P.11. Understand, predict, and interpret the effects of the parameters a, ω, b, and c on the graph of $y = a\sin(\omega(x - b)) + c$; do the same for the cosine and tangent. Use to model periodic processes.

PCT.P.12. Translate among geometric, algebraic, and parametric representations of curves. Apply to the solution of problems.

Example: The following is the parametric representation for a curve in the (x, y) plane:

$x(t) = 3\sin(2t)$ *and* $y(t) = 3\cos(2t)$ *for* $0 \leq t \leq \pi$

Show that this curve is a circle with radius 3.

(See also PCT.P.9)

Example: Change the parametric representation from the last example slightly and assume that

$x(t) = 5\sin(2t)$ *and* $y(t) = 3\cos(2t)$ *for* $0 \leq t \leq \pi$

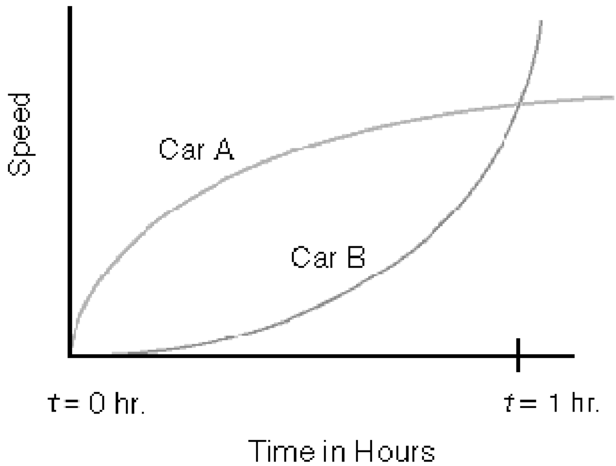
What sort of curve is described by the new equations? Justify your answer.

(See also PCT.P.9)

PATTERNS, RELATIONS, AND ALGEBRA INDICATORS (CONTINUED)

PCT.P.13. Relate the slope of a tangent line at a specific point on a curve to the instantaneous rate of change. Explain the significance of a horizontal tangent line. Apply these concepts to the solution of problems.

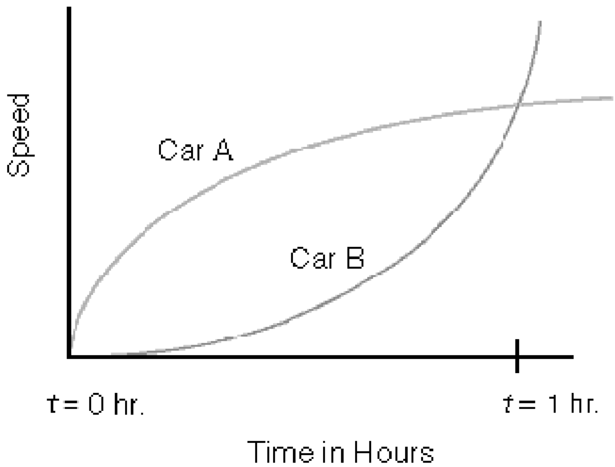
Example: Use the following graph in answering the questions below.



1. *State the relationship between the position of car A and that of car B at* t *= 1 hr. Explain.*
2. *State the relationship between the velocity of car A and that of car B at* t *= 1 hr. Explain.*
3. *State the relationship between the acceleration of car A and that of car B at* t *= 1 hr. Explain.*
4. *How are the positions of the two cars related during the time interval between* t *= 0.75 hr. and* t *= 1 hr.? (That is, is one car pulling away from the other?) Explain.*

PCT.P.14. Approximate areas under a curve.

Example: Assume that both cars reach 60mph at the 1 hour point on the graph. Explain how to use the graph to estimate the total distance traveled by the cars in one hour.



PCT.P.15. Demonstrate an understanding of the binomial theorem and use it in the solution of problems.

PCT.P.16. Identify maximum and minimum values of functions in simple situations. Apply to the solution of problems.

GEOMETRY INDICATORS

PCT.G.1. Demonstrate an understanding of the laws of sines and cosines. Use the laws to solve for the unknown sides or angles in triangles. Determine the area of a triangle given the length of two adjacent sides and the measure of the included angle.

Example: A stabilizing wire (guy wire) runs from the top of a 60 foot tower to a point 15 feet down the hill (measured on the slant) from the base of the tower. If the hill is inclined 11 degrees from the horizontal, how long does the wire need to be?

(See also PCT.G.3)

Example: Find the area enclosed by the triangle with one side of length 2, a second side of length $\sqrt{3}/2$ *, and an angle* $\theta = 30°$ *between these two sides.*

PCT.G.2. Use vectors to solve problems. Describe addition of vectors, multiplication of a vector by a scalar, and the dot product of two vectors, both symbolically and geometrically. Use vector methods to obtain geometric results.

PCT.G.3. Apply properties of angles, parallel lines, arcs, radii, chords, tangents, and secants to solve problems.

MEASUREMENT INDICATORS

PCT.M.1. Describe the relationship between degree and radian measures, and use radian measure in the solution of problems, particularly problems involving angular velocity and acceleration.

Example: In one hour, the minute hand on a clock moves through a complete circle, and the hour hand moves through 1/12 of a circle. Through how many radians do the minute and the hour hand move between 1:00 p.m. and 6:45 p.m. on the same day?

PCT.M.2. Use dimensional analysis for unit conversion and to confirm that expressions and equations make sense.

Example: Suppose a situation calls for determining the mass of a bar of gold bullion in the shape of a rectangular prism whose length, width, and height are measured as 27.9 centimeters, 10.2 centimeters, and 6.4 centimeters, respectively. Knowing that the density is 19,300 kilograms per cubic meter, compute the mass as follows:

Mass = (density) • (volume)

$$= \left(19{,}300\frac{kg}{m^3}\right)\bullet\left(\frac{1}{10^6}\frac{m^3}{cm^3}\right)\bullet\left(182.312cm^3\right)$$

$$= 35.1513216\ kg$$

(Students need to understand that reporting the mass with this degree of precision would be misleading because it would suggest a degree of accuracy far greater than the actual accuracy of the measurement. Since the lengths of the edges are reported to the nearest tenth of a centimeter, the measurements are precise only to 0.05 centimeter. That is, the edges could actually have measures in the intervals 27.9 ± 0.05, 10.2 ± 0.05, and 6.4 ± 0.05. If students calculate the possible maximum and minimum mass, given these dimensions, they will see that at most one decimal place in accuracy is justified. As suggested by the example above, units should be reported along with numerical values in measurement computations.)

DATA ANALYSIS, STATISTICS, AND PROBABILITY INDICATORS

PCT.D.1. Design surveys and apply random sampling techniques to avoid bias in the data collection.

Example: A consulting company develops a survey to measure student attitudes towards a new computer-based math tutoring system. The company creates a web page with the survey form and the survey is completed on-line by 1,200 high school students. The company reports that 92% of the responding students favor the computer-based system. Is there reason to doubt the high level of support for the system?

PCT.D.2. Apply regression results and curve fitting to make predictions from data and select appropriate functions as models.

PCT.D.3. Compare the results of simulations (e.g., random number tables, random functions, and area models) with predicted probabilities.

Example: You use a spreadsheet program to simulate 10,000 rolls of a fair die and obtain the following results:

Die Outcome	1	2	3	4	5	6
Number of Outcomes	1650	1701	1690	1633	1677	1649

How do these results compare with the probabilities for each outcome?